Chap 2. 2.97b. 2.98b. 2.95

2.95

Given: a structural system modeled with 5 point masses in space. coordinates with unit m

0.4 kg – (1,0,0) ; 0.4 kg – (1,1,0)

0.4 kg – (2,1,0) ; 0.4 kg – (2,0,0)

1.0 kg – (1.5, 1.5, 3)

Find: center of mass

It's a discrete system $\vec{r}_{cm} = \frac{\Sigma M_i \vec{r}_i}{M_{tot}}$

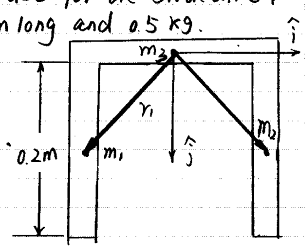
$m_{tot} = \sum_{i=1}^{5} m_i = 4 \times 0.4\,kg + 1.0\,kg = 2.6\,kg$

$$\Sigma m_i \vec{r}_i = 0.4\,kg(1m\hat{i}) + 0.4\,kg(1m\hat{i} + 1m\hat{j}) + 0.4\,kg(2m\hat{i} + 1m\hat{j}) + 0.4\,kg(2m\hat{i}) + 1.0\,kg(1.5m\hat{i} + 1.5m\hat{j} + 3m\hat{k}) = 3.9\,kgm\,\hat{i} + 2.3\,kgm\hat{j} + 3\,kgm\,\hat{k}$$

$$\Rightarrow \vec{r}_{cm} = \frac{\Sigma \vec{r}_i m_i}{m_{tot}} = \frac{(3.9\hat{i} + 2.3\hat{j} + 3\hat{k})\,m\,kg}{2.6\,kg} = 1.5\hat{i} + 0.88\hat{j} + 1.15\hat{k} \quad m$$

$$\boxed{\vec{r}_{cm} = 1.5\hat{i} + 0.88\hat{j} + 1.15\hat{k} \ (m)}$$

2.97b. Find the center of mass for the structure, all the bars are 0.2 m long and 0.5 kg.



Solution:

choose the coordinates as illustrated in the figure. then:

$\vec{r}_1 = -0.1\hat{i} + 0.1\hat{j}$ m

$\vec{r}_2 = 0.1\hat{i} + 0.1\hat{j}$ m

(if we didn't ignore the thickness of the bar h, then h should be counted for $\vec{r}_1$ and $\vec{r}_2$)

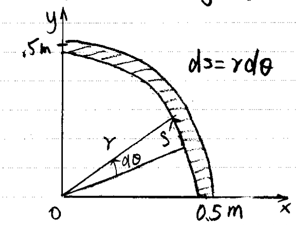
$\vec{r}_3 = \vec{0}$

$M_{tot} = 3 \times 0.5\,k = 1.5\,kg$

$$\Rightarrow \vec{r}_{cm} = \frac{\Sigma m_i \vec{r}_i}{M_{tot}} = \frac{0.5\,kg(-0.1m\hat{i} + 0.1m\hat{j}) + 0.5\,kg(0.1m\hat{i} + 0.1m\hat{j})}{1.5\,kg} = 0.067\,m\hat{j}$$

$$\boxed{\vec{r} = 0.067\,\hat{j} \quad (m)}$$

2.98b. Find the center of mass of the following object.



Soln:

It's a continous system:

$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{m_{tot}}$

• Finding $\int \vec{r}\,dm$

$dm = \rho\,ds = \rho\,r\,d\theta$

$\vec{r} = (r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}) = r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$r = 0.5$ m

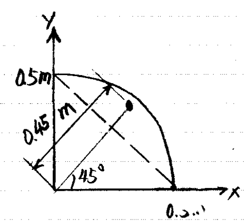
$$\Rightarrow \int \vec{r}\,dm = r^2\rho \int_0^{\frac{\pi}{2}} (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\,d\theta = r^2\rho\left\{+\sin\theta\Big|_0^{\frac{\pi}{2}}\hat{i} - \cos\theta\Big|_0^{\frac{\pi}{2}}\hat{j}\right\} = r^2\rho\{\hat{i} - (0-1)\hat{j}\} = r^2\rho(\hat{i}+\hat{j})$$

$$M_{tot} = \int dm = \int_0^{\frac{\pi}{2}} \rho\,r\,d\theta = r\rho\,\theta\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}\pi\rho r$$

$$\Rightarrow \vec{r}_{cm} = \frac{\int \vec{r}\,dm}{\int dm} = \frac{r^2\rho(\hat{i}+\hat{j})}{\frac{1}{2}\pi\rho r} = \frac{1}{\pi}(\hat{i}+\hat{j}) \ m = 0.32(\hat{i}+\hat{j}) \ m$$

$$\boxed{\vec{r}_{cm} = 0.32(\hat{i}+\hat{j}) \ m}$$

$|\vec{r}_{cm}| = 0.45$ m



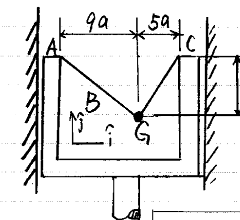
Chap. 3: 3.1a, 3.2, 3.10, 3.18, 3.39

3.1a: How does one know what forces and moments to use in the statics force balance and moment balance eqns?

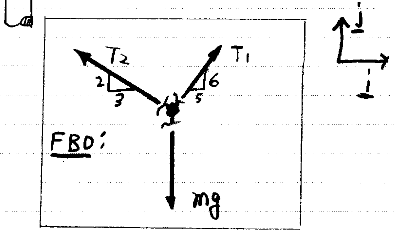
Soln:

By drawing a free body diagram. (Exactly the forces & moments on the FBD are used.)

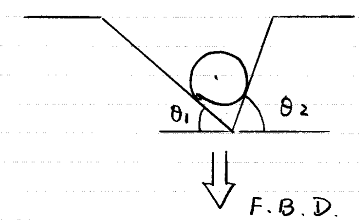
3.2. Draw the F.B.D. for the structure

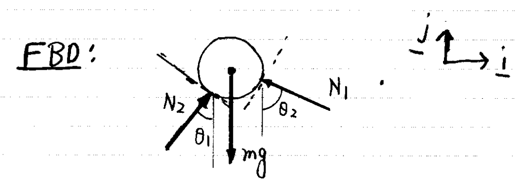


Although the structure is complecate, there're only three forces acted at mass G.

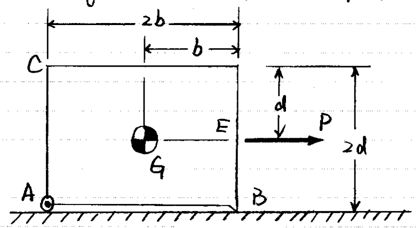


3.10. Draw the F.B.D of the disk. There's gravity and negligible friction

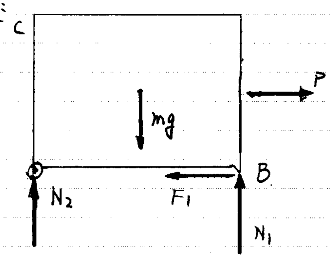




3.18. Draw the F.B.D, assuming the block is sliding to the right with coefficient of friction μ at B.

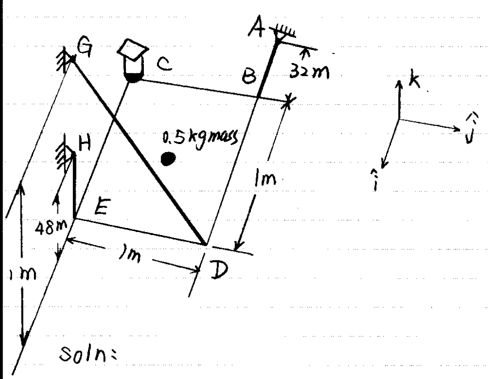


F.B.D:



At point A, we have no friction force, because it's a ideal wheel

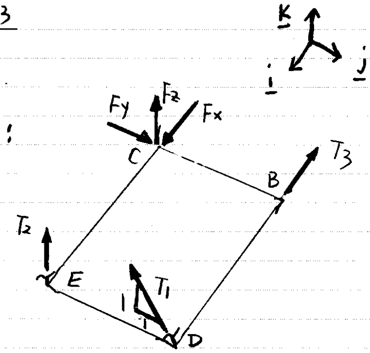
3.39. Draw the F.B.D. The shelf is in a rocket with $10\ m/s^2$ in the $\hat{k}$ direction.



Soln:

Cont. 3.39

FBD:



F.B.D is illustrated above.

At point D, we have all the three components of the ball and socket constraint, but no moment.

All the three Ts come from the tension in string. They're all along the strings.

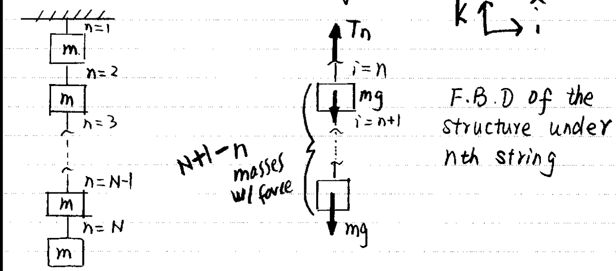
There's acceleration of $10\ m/s^2$ in $\hat{k}$ direction, but we don't add it to the F.B.D. so there's no gravity for mass at the center.

Chap. 4 : 4.2, 4.4, 4.3

4.2

Given: N small blocks each of mass m hang vertically connected by N inextensible string

Find: tension T_n in string n



Soln:

From the F.B.D of the structure under nth string

$\Sigma \vec{F} = 0$

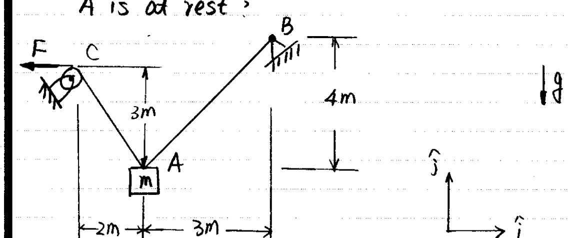
$\Rightarrow T_n \hat{k} + (N+1-n)\, mg(-\hat{k}) = \vec{0}$

$\Rightarrow [T_n \hat{k} + (N+1-n)\, mg(-\hat{k})] \cdot \hat{k} = \vec{0}$

$\Rightarrow T_n = (N+1-n)\, mg$

*: Total of blocks under nth string is $(N+1-n)$

4.4. What force should be applied to the end of the string over the pulley at C so that the mass at A is at rest?

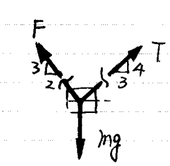


Soln:

Draw the F.B.D of mass m:

$\Sigma \vec{F} = 0$

FBD:



$\vec{F}_1 = F \hat{\lambda}_{AC}(N) = F \dfrac{-2\hat{i}+3\hat{j}}{\sqrt{2^2+3^2}}$

$= F \dfrac{1}{\sqrt{13}}(-2\hat{i}+3\hat{j})$

$\vec{F}_2 = T \hat{\lambda}_{AB}(N) = T \dfrac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}} N = T \dfrac{1}{5}(3\hat{i}+4\hat{j})$

Cont. 4.4

$\vec{F_3} = mg(-\hat{j})$

$\Rightarrow \Sigma\vec{F} = F\cdot\frac{1}{\sqrt{13}}(-2\hat{i}+3\hat{j}) + T\,\frac{1}{5}\,(3\hat{i}+4\hat{j}) - mg\hat{j} = \vec{0}$

$\Rightarrow$

$(-\frac{2F}{\sqrt{13}} + \frac{3}{5}T)\,\hat{i} + (\frac{3F}{\sqrt{13}} + \frac{4}{5}T - mg)\,\hat{j} = 0$

$\Rightarrow$

$$\begin{cases} \frac{3}{5}T - \frac{2}{\sqrt{13}}F = 0 \Rightarrow T = \frac{2}{\sqrt{13}}\frac{5}{3}F = \frac{10}{3}\frac{1}{\sqrt{13}}F \\ \frac{4}{5}T + \frac{3}{\sqrt{13}}F - mg = 0 \end{cases}$$

plugging $T = \frac{10}{3\sqrt{13}}F$ to the second eqn:

$\frac{4}{5}\,\frac{10}{3}\,\frac{1}{\sqrt{13}}F + \frac{3}{\sqrt{13}}F = mg$

$\Rightarrow F = \frac{3\sqrt{13}}{17}mg$

$\boxed{F = \frac{3}{17}\sqrt{13}mg}$

method 2: use $\Sigma F_x = 0$, $\Sigma F_y = 0$
if you don't like the vector, you can simply set the components be 0

$\Sigma F_x = T\cdot\cos\beta - F\cos\theta$
$= T\cdot\frac{3}{\sqrt{3^2+4^2}} - F\frac{2}{\sqrt{3^2+2^2}}$
$= \frac{3}{5}\cdot T - \frac{2}{\sqrt{13}}F$ (1)
$= 0$

$\Sigma F_y = T\sin\beta + F\sin\theta - mg$
$= T\frac{4}{5} + F\frac{3}{\sqrt{13}} - mg$
$= 0$ (2)

From (1) $\Rightarrow T = \frac{10}{3}\frac{1}{\sqrt{13}}F$, plugging it into (2)

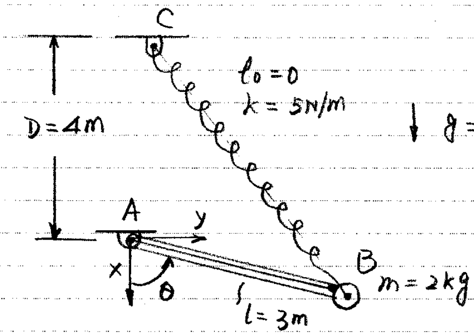
$\Rightarrow \frac{10}{3}\frac{1}{\sqrt{13}}\frac{4}{5}F + \frac{3}{\sqrt{13}}F = mg$

$\Rightarrow F = \frac{3\sqrt{13}}{17}mg$

4.3.

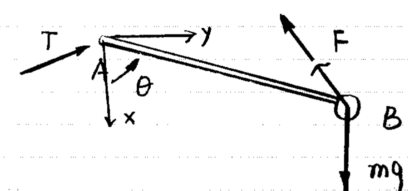
Given: $k = 5\ N/m$, $g = 10\ N/m$

Find: θ for static equilibrium



Method 1: study structure AB:

F.B.D:



We have:

$\Sigma\vec{M}_A = 0$

$\Sigma\vec{M}_A = \vec{r}_{AB}\times\vec{F} + \vec{r}_{AB}\times mg(\hat{i})$

$\vec{F} = k\Delta L\,\hat{\lambda}_{BC}$ $\qquad \vec{r}_C = -4\hat{i}\ m$
$= k(l-l_0)\,\lambda_{BC}$
$= k\,l\,\lambda_{BC}$ $\qquad l_0 = 0$
$= k|\vec{r}_{BC}|\,\lambda_{BC}$
$= k|\vec{r}_{BC}|\frac{\vec{r}_{BC}}{|\vec{r}_{BC}|}$
$= k\,\vec{r}_{BC}$
$= k(\vec{r}_C - \vec{r}_B)$

$\Rightarrow$

$\Sigma\vec{M}_A = \vec{r}_B\times k(\vec{r}_C - \vec{r}_B) + \vec{r}_B\times(mg\hat{i})$
$= \vec{r}_B\times k\vec{r}_C + \vec{r}_B\times mg\hat{i}$
$= \vec{r}_B\times(k4m\hat{i} + mg\hat{i})$
$= \vec{r}_B\times(-20N\hat{i} + 20N\hat{i})$
$= 0$

$\Rightarrow$ $\boxed{\theta \text{ is arbitrary!}}$

Cont. 4.3

Also we need to check $\Sigma \vec{F}=0$ to get equilibrium. By solving it, you can find that $\vec{T}$ is along the direction of the bar and $\Sigma \vec{F}=0$ at any angle.

Method 2: study the mass point and let $\Sigma \vec{F}=0$

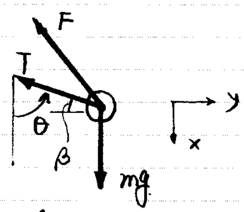
($\Sigma \vec{M}=0$ is already automatically satisfied)

$\vec{T}$ is along the bar (T possible <0) because it's a 2-force member.

Soln:

Draw the F.B.D of mass $m = 2kg$

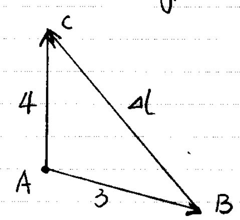
For static equilibrium:



$\Sigma \vec{F} = 0$

$$\vec{T} = T\hat{\lambda}_{BA}$$
$$= T(-\sin\beta\,\hat{i} - \cos\beta\,\hat{j})$$
$$= T(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$\vec{F} = K\Delta L\,\hat{\lambda}_{BC}$$



- Finding ΔL

$$\Delta L = |\vec{r}_{BC}|$$
$$= |\vec{r}_{BC}|$$
$$= |\vec{r}_{AC} - \vec{r}_{AB}|$$

$$\vec{r}_{AC} = -4\hat{i} \quad m$$

$$\vec{r}_{AB} = 3\hat{\lambda}_{AB} \quad m$$
$$= 3(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \quad m$$

$$\Rightarrow \vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB} = -4\hat{i} - 3(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \quad m$$
$$= (-4-3\cos\theta)\hat{i} - 3\sin\theta\,\hat{j} \quad m$$

$$\Rightarrow \Delta L = |\vec{r}_{BC}| = \sqrt{(3\sin\theta)^2 + (-4-3\cos\theta)^2} \quad m$$
$$= \sqrt{25-24\cos\theta} \quad m$$

- Finding $\hat{\lambda}_{BC}$

$$\hat{\lambda}_{BC} = \frac{\vec{r}_{BC}}{|\vec{r}_{BC}|} = \frac{1}{\sqrt{25-24\cos\theta}}\left[(-4-3\cos\theta)\hat{i} - 3\sin\theta\,\hat{j}\right]$$

Cont. 4.3

- Finding F in terms of θ

$$\vec{F} = K\Delta L\,\hat{\lambda}_{BC} \qquad K = 5\ N/m$$
$$= 5\sqrt{25-24\cos\theta}\,\frac{1}{\sqrt{25-24\cos\theta}}\left((-4-3\cos\theta)\hat{i} - 3\sin\theta\,\hat{j}\right)(N)$$
$$= \left[(-20-15\cos\theta)\hat{i} - 15\sin\theta\,\hat{j}\right] \quad (N)$$

- Using $\Sigma \vec{F} = \vec{0}$ to find θ

$$\Sigma \vec{F} = \vec{F} + \vec{T} + mg\,\hat{i}$$
$$= [(-20-15\cos\theta)\hat{i} - 15\sin\theta\,\hat{j}] \quad N$$
$$+[T(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})]$$
$$+ mg\,\hat{i}$$

$$= (mg - (20-15\cos\theta)N - T\cos\theta)\hat{i} - (T\sin\theta + 15\sin\theta N)\hat{j}$$
$$= \vec{0}$$

$$\Rightarrow \begin{cases} mg - (20-15\cos\theta)N - T\cos\theta = 0 & (1) \\ T\sin\theta + 15\sin\theta N = 0 & (2) \end{cases}$$

From (2) $\Rightarrow$ $T = -15$ (N) (means the direction should be opposite)

plugging $T=-15N$ into (1)

$\Rightarrow$

$$mg - (20 - 15\cos\theta + 15\cos\theta)N = 0$$

$\Rightarrow$ $2\times10-20=0$ automatically satisfied!

Thus the result is independent of θ.

$$\boxed{\text{Equilibrium is independent of } \theta\,!}$$